EU applications

~~Insurance~~

~~the DM with strict risk aversion~~

W initial wealth

D - possible loss with prob. π

can buy insurance that costs q per unit and pays 1 if loss occurs

the DM is deciding how many units of insurance, α, he wishes to buy

Insurance: $L = (w - \alpha q, 1-\pi;$
$w - \alpha q - D + \alpha; \pi)$

$$E(L) = (1-\pi)(w-\alpha q) + \pi(w - \alpha q - D + \alpha)$$
$$= w - \alpha q - \pi(D-\alpha)$$

$$\max_{\alpha} Eu(L) = (1-\pi)\,u(w-\alpha q) + \pi\,u(w-\alpha q - D + \alpha)$$

FOC: $-q(1-\pi)\,u'(w-\alpha^* q) + \pi(1-q)\,u'(w + (1-q)\alpha^* - D) =$

assuming $\alpha^* > 0$.

suppose that the insurance is actuarially ~~safe~~ fair $q = \pi$

then FOC becomes:

$$u'(w + (1-q)\alpha^* - D) = u'(w - \alpha^* q)$$

since u' is strictly decreasing by strict risk aversion we must have

$$w + (1-q)\alpha^* - D = w - \alpha^* q$$

or

$$\alpha^* = D \quad \text{full insurance}$$

if insurance is actuarily fair, the a strict risk averter will choose full insurance

Portfolio choice

2 assets: safe and risky

→ safe: has a return of 1 dollar per dollar invested

→ risky: random return of $z \in [a, b]$ dollars with distribution F. Assume that $E(z) > 1$

the DM with utility $u(\cdot)$ has initial wealth w and invests α in the risky asset

end of period wealth:

$$(w-\alpha)\cdot 1 + \alpha \cdot z = w + (z-1)\alpha$$

$$\max_{\alpha} V(\alpha, w) = \int_a^b u(w + (z-1)\alpha)\, dF(z)$$

$$\text{s.t. } 0 \leq \alpha \leq w$$

α^* is the solution

1) show that $\alpha^* > 0$

2) see how α^* varies with levels of risk aversion and with wealth

KT FOCs

$$\phi(\alpha^*, w) = \int_a^b (z-1)\, u'(w + (z-1)\alpha^*)\, dF(z) \begin{cases} < 0 & \text{if } \alpha^* = 0 \\ = 0 & \text{if } 0 < \alpha^* < w \\ > 0 & \text{if } \alpha^* = w \end{cases}$$

SOC:

$$\int_a^b (z-1)^2 u''(w + (z-1)\alpha^*)\, dF(z) < 0$$

since $(z-1)^2 \geq 0$ and $u'' < 0$

Prop. If a risk is actuarily favorable, then any risk averter will always accept at least a small amount of it.

Proof Note that $\phi(0, w) = \int_a^b (z-1)\, u'(w)\, dF(z) > 0$

since $E(z) > 1$

Hence $\alpha^* = 0$ cannot satisfy the FOC

Prop Consider 2 individuals with the same level of initial wealth. If individual 2 is strictly more RA than an individual 1, ind. 2 will invest less wealth in a risky asset i.e. $\alpha_1^* > \alpha_2^*$

proof

consider 2 inds with $u_1(\cdot)$ and $u_2(\cdot)$ where 2 is strictly more RA than 1. Suppose that both have the same level of wealth w and that $\alpha_1^* < w$ and $\alpha_2^* < w$

$$\phi_1(\alpha_1^*) = \int_a^b (z-1)\, u_1'(w+(z-1)\alpha_1^*)\, dF(z) = 0$$

$$\phi_2(\alpha_2^*) = \int_a^b (z-1)\, u_2'(w+(z-1)\alpha_2^*)\, dF(z) = 0$$

since $u'' < 0$, ϕ_1, ϕ_2 are decreasing functions, to prove that $\alpha_1^* > \alpha_2^*$, it is sufficient to show that $\phi_2(\alpha_1^*) < \cancel{\phi}.0 = \phi_2(\alpha_2^*)$

since ind. 2 is strictly more risk averse that 1, there exists a str. concave function $\psi(\cdot)$ s.t.

$$u_2(x) = \psi(u_1(x))$$

$$V_2(\alpha) = \int_a^b \psi(u_1(w+(z-1)\alpha))\, dF(z)$$

differentiating wrt α

$$\phi_2(\alpha) = \int_a^b (z-1)\, \psi'(u_1(w+(z-1)\alpha))\, u_1'(w+(z-1)\alpha)\, dF(z)$$

which at α_1^* is

$$\int_a^b (z-1)\, \psi'(u_1(w+(z-1)\alpha_1^*))\, u_1'(w+(z-1)\alpha_1^*)\, dF(z)$$

where $\psi'(u_1(w+(z-1)\alpha_1^*))$ is positive and decreasing <u>in z</u>

Compare

$$\phi_2(\alpha_1^*) = \int_a^b (z-1)\,\varphi'(u_1(w+(z-1)\alpha_1^*))\,u_1'(w+(z-1)\alpha_1^*)\,dF(z)$$

to

$$\phi_1(\alpha_1^*) = \int_a^b (z-1)\,u_1'(w+(z-1)\alpha_1^*)\,dF(z) = 0$$

the term $\varphi'(u_1(w+(z-1)\alpha_1^*))$ is higher for z's less than 1 than for z's greater than 1. So it biases the weights from $u_1'(w+(z-1)\alpha_1^*)$ which are perfectly balanced in individual 1's FOC.

Therefore $\phi_2(\alpha_1^*) < 0$. Hence $\alpha_1^* > \alpha_2^*$