SummerLab
Games & Decisions

Part II: Explaining departures from Nash Equilibrium

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The motivating example – the traveler's dilemma game

An airline loses 2 identical suitcases belonging to 2 different travelers.

The airline manager separates the travelers and asks them to write down the value of their case between \$2 and \$100.

If both write down the same amount, each gets this amount.

If one amount is smaller, then each of them will get this amount with a bonus/malus:

- The traveler who chose the smaller amount will get \$2 extra
- The other traveler will have to pay \$2 penalty.

The traveler's dilemma game

What strategy should both travelers follow to decide the value they should write down?

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- Intuitively: close to \$100
- Nash Equilibrium strategy: \$2

How about if the bonus/malus amount equals say \$150?

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What strategy should both travelers follow to decide the value they should write down?

- Intuitively: close to $100
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How about if the bonus/malus amount equals say $150?

- Intuitively: close to $2
- Nash Equilibrium strategy: $2

The game presented above was first proposed by Basu (1994) and is called the ***traveler's dilemma***.

Some motivating questions

Why is Nash Equilibrium strategy intuitive in some cases and not in other?

How can we explain departures from the game theory predictions?

When people do not play according to the game theory predictions, are they rational or irrational?

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Motivating article

Jacob Goeree and Charles Holt *Ten little treasures of game theory and ten intuitive contradictions*, AER, 2001

- Lab data for games played once.
- For each game two payoff structures:
  - the treasure in which the observed behavior agrees with the NE prediction.
  - another one which produces a striking inconsistency between the two.

We shall focus only on **static games of complete information**.

Outline of the lecture

1. Some theory
2. Treasures of game theory
3. Hypothesis
4. The Game of Rows mobile app

Consider a game Γ in strategic form:

1. The set of players N
2. For each player $i \in N$ a set of actions A_i
3. For each player $i \in N$ a payoff function $u_i : A \to \mathbb{R}$

Notation:

- A set of all players profiles of actions is denoted by $A \equiv \times_{i \in N} A_i$ with a typical element denoted by a.
- A set of all players but player i profiles of actions is denoted by $A_{-i} \equiv \times_{j \in N \setminus i}$ with a typical element denoted by a_{-i}.

Nash Equilibrium in pure strategies

Definition

Nash equilibrium in pure strategies is a profile of actions $a^* \in A$ such that:

$$a_i^* \in \arg\max_{a_i \in A_i} u_i(a_i, a_{-i}^*), \quad \forall a_{-i} \in A_{-i}, \ \forall i \in N \tag{1}$$

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Mixed strategies

Definition

Given any strategic form game Γ, a randomized strategy for any player i is a probability distribution over A_i. Let $\Delta(A_i)$ denote the set of all possible randomized strategies for player i. The set of all randomized strategy profiles will be denoted by $\Delta(A) = \times_{i \in N} \Delta(A_i)$. It must be that:

$$\sum_{a_i \in A_i} \sigma_i(a_i) = 1, \quad \forall i \in N$$

We will write $\sigma \equiv (\sigma_i)_{i \in N}$, where $\sigma_i \equiv (\sigma_i(a_i))_{a_i \in A_i}$, for each i. For any randomized strategy profile σ, let $u_i(\sigma)$ denote the expected payoff that player i would get when the players independently choose their pure strategies according to σ:

$$u_i(\sigma) = \sum_{a \in A} \left(\prod_{j \in N} \sigma_j(a_j) \right) u_i(a), \quad \forall i \in N$$

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For any $\sigma_i' \in \Delta(A_i)$, we denote (σ_i', σ_{-i}) the randomized strategy profile in which the i-th component is σ_i' and all other components are as in σ. Thus:

$$u_i(\sigma_i', \sigma_{-i}) = \sum_{a \in A} \left(\prod_{j \in N \setminus i} \sigma_j(a_j) \right) \sigma_i'(a_i) u_i(a)$$

Definition

A randomized strategy profile $\sigma^* \in \Delta(A)$ is a Nash equilibrium of Γ if the following holds:

$$\sigma_i^* \in \arg \max_{\sigma_i \in \Delta(A_i)} u_i(\sigma_i, \sigma_{-i}^*), \quad \forall i \in N \tag{2}$$

Common knowledge of rationality

Definition

There is ***common knowledge*** *of p in a group of agents G when all the agents in G know p, they all know that they know p, they all know that they all know that they know p, and so on ad infinitum.*

The crucial **assumption** underlying the Nash Equilibrium is that there is common knowledge among players in the game about:

- the game and all its components,
- rationality of all players.

We shall question this assumption.

Definition

*The **rationalizable set of actions** (Bernheim, 1984 and Pearce, 1984) can be computed as follows:*

1. *Start with the full action set for each player.*
2. *Remove all actions which are never a best reply to any belief about the opponents' actions*
   - *No rational player will choose such actions.*
3. *Remove all actions which are never a best reply to any belief about the opponents' remaining actions*
   - *Each player knows that the other players are rational.*
4. *Continue the process until no further actions are eliminated.*
5. *In a game with finitely many actions, this process always terminates and leaves a non-empty set of actions for each player.*

Rationalizability

Players respond optimally to some belief about their opponents' actions, but Nash equilibrium requires that these beliefs be correct while rationalizability does not.

The general idea is to provide the weakest constraints on players while still requiring that players are rational and this rationality is common knowledge among the players.

Mixed strategy equilibrium calculation

Robert \ Cindy	high	low
high	2 2	1 -3
low	-3 1	1 1

- Let p (q) denote the probability of Robert (Cindy) choosing "high".

- **Robert's best response function**
- Expected payoff from choosing:
  - high: $2q - 3(1-q) = -3 + 5q$
  - low: 1
- If $-3 + 5q > 1$ or $q > 4/5$, Robert should choose high, i.e. $p = 1$.
- If $-3 + 5q < 1$ or $q < 4/5$, he should choose low, i.e.$p = 0$.
- If $-3 + 5q = 1$ or $q = 4/5$, then he is indifferent $p \in [0,1]$.

Robert's best response correspondence:

$$BR_R(q) = \begin{cases} \{1\}, & \text{if } q > 4/5 \\ [0,1], & \text{if } q = 4/5 \\ \{0\}, & \text{if } q < 4/5 \end{cases}$$

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- **Cindy's best response function**
- Expected payoff from choosing:
  - high: $2p - 3(1-p)$
  - low: 1
- If $2p - 3(1-p) > 1$ or $p > 4/5$, Cindy should choose high, i.e. $q = 1$
- If $2p - 3(1-p) < 1$ or $p < 4/5$, she should choose low, i.e. $q = 0$
- If $2p - 3(1-p) = 1$ or $p = 4/5$, then she is indifferent, i.e. $q \in [0,1]$

Cindy's best response correspondence:

$$BR_C(p) = \begin{cases} \{1\}, & \text{if } p > 4/5 \\ [0,1], & \text{if } p = 4/5 \\ \{0\}, & \text{if } p < 4/5 \end{cases}$$

Note that in calculating mixed strategy for one player, one takes into account only payoffs of his/her opponent, not his/her own.

Treasures of game theory

- simple coordination game,
- the traveller's dilemma,
- a matching pennies game,
- a coordination game with a secure outside option,
- a minimum effort coordination game,
- the Kreps game

Simple coordination game

Consider two games of a "choose-an-effort" variety.

Game A

	L	R
T	$\underline{2}, \underline{2}$	$-3, 1$
B	$1, -3$	$\underline{1}, \underline{1}$

Game B

	L	R
T	$\underline{5}, \underline{5}$	$0, 1$
B	$1, 0$	$\underline{1}, \underline{1}$

There are two Nash equilibria in pure strategies (T, L), (B, R) in each of the games and the mixed strategy equilibrium:

- $(.8T + .2B, .8L + .2R)$ in game A
- $(.2T + .8B, .2L + .8R)$ in game B.

Assuming people play the mixed strategy, we should most often (64% of the time) observe (T, L) in game A, and (B, R) in game B.

Experimental evidence shows exactly the opposite: (B, R) is most frequent in game A and (T, L) in game B.

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The one-shot traveller's dilemma game

Let's consider the travellers' dilemma with two players and action space $A_i = \{180, 181, ..., 300\}$ for each $i \in N = \{1, 2\}$. Payoffs are the following:

$$u_i(a_i', a_j') = \min(a_i', a_j') + P(a_i', a_j'), \quad i \neq j, \quad i, j \in N,$$

$$\text{where} \quad P(a_i', a_j') = \begin{cases} P & \text{if} \quad a_i' < a_j' \\ 0 & \text{if} \quad a_i' = a_j' \\ -P & \text{if} \quad a_i' > a_j' \end{cases}, \quad \text{where} \quad P \in \mathbb{Z}^+$$

Analysis

When $P = 0$ and for $P = 1$, any profile of strategies for which $a_i = a_j, \quad i \neq j$ is a pure strategy Nash equilibrium.

When $P > 1$, note that given any strategy of the opponent it is optimal to underbid her by \$1. So bidding \$300 is never a best response to any belief.

Since players are rational and know that their opponent is rational, they can delete this action from their strategy space.

But then \$299 is never a best response to any belief. So it can be deleted as well.

Continuing this way, the only strategy pair, that survives this iterated procedure is $(180, 180)$, i.e. the unique rationalizable strategy profile.

How about experimental evidence, R denotes the bonus/malus

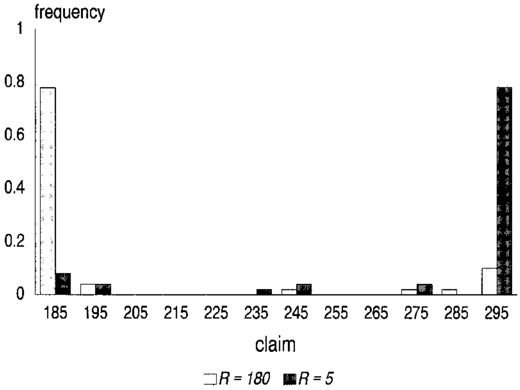

FIGURE 1. CLAIM FREQUENCIES IN A TRAVELER'S DILEMMA FOR $R = 180$ (LIGHT BARS) AND $R = 5$ (DARK BARS)

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The matching pennies games (in brackets: mixed NE in BLACK, experimental evidence in RED)

Game A

Payoff table

	L (.50)/(.48)	R
T (.50)/(.48)	$\underline{80}, 40$	$40, \underline{80}$
B	$40, \underline{80}$	$\underline{80}, 40$

Game B

Payoff table

	L (.13)/(.16)	R
T (.50)/(.96)	$\underline{320}, 40$	$40, \underline{80}$
B	$40, \underline{80}$	$\underline{80}, 40$

Game C

Payoff table

	L (.91)/(.80)	R
T (.50)/(.08)	$\underline{44}, 40$	$40, \underline{80}$
B	$40, \underline{80}$	$\underline{80}, 40$

A coordination game with a secure outside option (in brackets: mixed NE in BLACK, $x = 0$ treatment in RED, $x = 400$ treatment in BLUE)

The following is the so called extended coordination game:

Payoff table

	L (.67)	M (.33)/(.84)/(.76)	R
T	$\underline{90}, \underline{90}$	$0, 0$	$x, 40$
B (.33)/ (.96)/(.64)	$0, 0$	$\underline{180}, \underline{180}$	$0, 40$

Since strategy R is dominated by a 50-50 combination of L and M, so it cannot be part of any Nash equilibrium. It is easy to see that the set of Nash equilibria of this game is the same irrespective of the value x

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A minimum effort coordination game

Consider the game of choosing an effort level, where $N = \{1, 2\}$, $A_i = \{110, 111, ..., 170\},\ i \in N$. The payoffs for a given profile of actions $(a'_i, a'_j) \in A$ are defined as follows:

$$u_i(a'_i, a'_j) = \min(a'_i, a'_j) - ca'_i$$

where $c \in (0, 1)$.
The set of Nash equilibria consists of all the pairs (a, a), where $a \in A_i$.

Experimental evidence

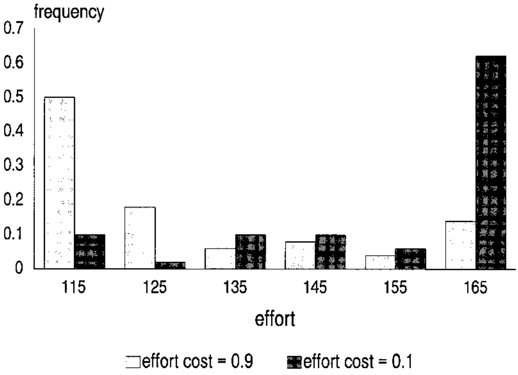

FIGURE 2. EFFORT CHOICE FREQUENCIES FOR A MINIMUM-EFFORT COORDINATION GAME WITH HIGH EFFORT COST (LIGHT BARS) AND LOW EFFORT COST (DARK BARS)

The Kreps game (in brackets mixed NE in BLACK and experimental data in BLUE)

The following is the so called Kreps game:

Payoff table

	L (.13)/(.26)	M (.08)	NN (.68)	R (.87)/(.00)
T (.49) /(.68)	$\underline{200}, \underline{50}$	$0, 45$	$10, 30$	$20, -250$
B (.51)/(.32)	$0, -250$	$\underline{10}, -100$	$\underline{30}, 30$	$\underline{50}, \underline{40}$

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Conjecture - part I

Common knowledge of rationality is a very strong assumption

It is violated if people:

- make mistakes
- do not pay attention
- are not sure about their opponent, his/her motivation, state of mind, etc.

Sometimes playing the NE strategy is risky, i.e. in case your opponent does not conform, you may loose a lot.

Maybe we should drop the assumption of common knowledge of rationality.

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Example: Choosing among multiple Nash Equilibria

Game A

	L	R
T	$\underline{2}, \underline{2}$	$-3, 1$
B	$1, -3$	$\underline{1}, \underline{1}$

Game B

	L	R
T	$\underline{5}, \underline{5}$	$0, 1$
B	$1, 0$	$\underline{1}, \underline{1}$

Observe that the mixed strategy is calculated based on your opponent's payoff only.

Note that if we drop the assumption of common knowledge of rationality, playing T (or L) in game A is **more risky** in game A than in game B

- in A you risk getting -3 instead of 1 and in B getting 0 instead of 1

We can capture this effect by treating the opponent as unpredictable nature.

Game A	s_1	s_2
T or L	2	-3
B or R	1	1

Game B	s_1	s_2
T or L	5	0
B or R	1	1

where s_1, s_2 are two states of the world.
Observe that the strategies T and L in the game A are more uncertain (giving the payoff of either 2 or -3) than the strategies B and R (giving the certain payoff of 1). Similarly, in the game B the strategies B and R are certain (giving the payoff of 1), whereas the strategies T and L are uncertain (giving the payoff of either 5 or 0).

Uncertainty

Uncertainty is modeled by having preferences defined over acts $f : S \to X$, where S is an exhaustive and exclusive set of states of nature and X is the set of consequences.

Frank Knight (1921) distinguished between:

- Uncertainty (no quantifiable information is available)
- Risk (exact probabilities are given)

Mark Machina prefers to use the terms:

- Objective uncertainty
- Subjective uncertainty

Beliefs (probabilities) are very subjective. Complete ignorance does not require probabilities. It is a good benchmark.

Rules for making decisions under complete ignorance

1. Maxmin rule (Abraham Wald)
2. Maxmax rule
3. Minmax regret rule (Leonard Savage)
4. Principle of insufficient reason (Jacob Bernoulli)
5. Hurwicz criterion

A simple choice problem

Consider the following choice problem:

	Crisis	No crisis
Safe	0	2
Medium	-3	6
Risky-Hedge	10	-6

A simple choice problem

Consider the following choice problem:

	Crisis	No crisis
Safe	0	2
Medium	-3	6
Risky-Hedge	10	-6

The investor is ignorant and he does not know the probability of Crisis/No Crisis.

MaxiMin

The investor may choose a strategy that maximizes the minimal payoffs across the states

The investor may choose a strategy that maximizes the minimal payoffs across the states

	C	NC	MaxMin
S	0	2	0
M	-3	6	-3
R-H	10	-6	-6

MaxiMin

The investor may choose a strategy that maximizes the minimal payoffs across the states

	C	NC	MaxMin
S	0	2	0
M	-3	6	-3
R-H	10	-6	-6

Drawback: pessimism

MaxMax

Alternatively, the investor may choose a strategy that maximizes the maximal payoffs across the states

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	C	NC	MaxiMax
S	0	2	2
M	−3	6	6
R-H	10	−6	10

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MaxMax

Alternatively, the investor may choose a strategy that maximizes the maximal payoffs across the states

	C	NC	MaxiMax
S	0	2	2
M	−3	6	6
R-H	10	−6	10

Drawback: optimism

Hurwicz α criterion

The investor may choose a strategy that maximizes a linear combination of Maximin and Maximax

Hurwicz α criterion

The investor may choose a strategy that maximizes a linear combination of Maximin and Maximax

	C	NC	Hurwicz
S	0	2	$\alpha \times 0 + (1-\alpha) \times 2$
M	-3	6	$\alpha \times (-3) + (1-\alpha) \times 6$
R-H	10	-6	$\alpha \times (-6) + (1-\alpha) \times 10$

Hurwicz α criterion

The investor may choose a strategy that maximizes a linear combination of Maximin and Maximax

	C	NC	Hurwicz
S	0	2	$\alpha \times 0 + (1-\alpha) \times 2$
M	-3	6	$\alpha \times (-3) + (1-\alpha) \times 6$
R-H	10	-6	$\alpha \times (-6) + (1-\alpha) \times 10$

Drawbacks: a) the pessimism coefficient is a free parameter, b) mixing of two optimal actions may not be optimal

Laplace

The investor may assume that each state is equally probable and choose a strategy that maximize expected value

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The investor may assume that each state is equally probable and choose a strategy that maximize expected value

	C	NC	Laplace
S	0	2	$\frac{1}{2} \times 0 + \frac{1}{2} \times 2 = 1$
M	-3	6	$\frac{1}{2} \times (-3) + \frac{1}{2} \times 6 = 1.5$
R-H	10	-6	$\frac{1}{2} \times (-6) + \frac{1}{2} \times 10 = 2$

Laplace

The investor may assume that each state is equally probable and choose a strategy that maximize expected value

	C	NC	Laplace
S	0	2	$\frac{1}{2} \times 0 + \frac{1}{2} \times 2 = 1$
M	-3	6	$\frac{1}{2} \times (-3) + \frac{1}{2} \times 6 = 1.5$
R-H	10	-6	$\frac{1}{2} \times (-6) + \frac{1}{2} \times 10 = 2$

Drawbacks: a) Subdividing states may change the optimal action, b) seems arbitrary: does not reflect lack of knowledge

Minimax regret

The investor may choose a strategy that minimizes a maximal regret across states

Minimax regret

The investor may choose a strategy that minimizes a maximal regret across states

	Payoff table		Regret table		
	C	NC	C	NC	Minimax regret
S	0	2	10	4	10
M	−3	6	13	0	13
R-H	10	−6	0	12	12

Drawback: violates independence of irrelevant alternatives

Minimax regret

Each of these rules have their drawbacks.

However, note that in the context of a game, violation of IIA in the case if minimax regret is not an issue.

Hence we postulate using this rule.

Conjecture - part II

Conjecture: People depart from the NE strategy if it is very risky to play it.

There is a trade-off between:

- What is achievable using strategic interaction (Nash equilibria)
- And what is risky in case others do not conform to the rationality principle (minimax regret strategies).

To predict what will actually be played in a given game one should weight the benefits of the NE stability and minimax regret safety.

Definition

Regret *from chosing* $a_i' \in A_i$ *given a profile of other players' actions* $a_{-i}' \in A_{-i}$ *is defined as:*

$$R_i(a_i', a_{-i}') = \max_{a_i \in A_i}[u_i(a_i, a_{-i}')] - u_i(a_i', a_{-i}') \tag{3}$$

Maximum regret *from choosing* $a_i' \in A_i$ *is then given by:*

$$\max_{a_{-i} \in A_{-i}} R_i(a_i', a_{-i}) \tag{4}$$

Minimax regret strategy *for a player* $i \in N$ *is an action* $a_i^* \in A_i$ *such that:*

$$a_i^* \in \arg\min_{a_i \in A_i} \left[\max_{a_{-i} \in A_{-i}} R_i(a_i, a_{-i}) \right] \tag{5}$$

Choosing among multiple Nash Equilibria - regret

Payoff table

	L	R
T	$\underline{2}, \underline{2}$	$-3, 1$
B	$1, -3$	$\underline{1}, \underline{1}$

Regret table

	L	R	max
T	$0, 0$	$4, 1$	4
B	$1, 4$	$0, 0$	1
max	4	1	

Payoff table

	L	R
T	$\underline{5}, \underline{5}$	$0, 1$
B	$1, 0$	$\underline{1}, \underline{1}$

Regret table

	L	R	max
T	$0, 0$	$1, 4$	1
B	$4, 1$	$0, 0$	4
max	1	4	

Max regret shows that the safe strategy is (B, R) in A and (T, L) in B.

Matching pennies

Game A

Payoff table

	L	R
T	$\underline{80}, 40$	$40, \underline{80}$
B	$40, \underline{80}$	$\underline{80}, 40$

Regret table

	L	R	max
T	$0, 40$	$40, 0$	40
B	$40, 0$	$0, 40$	40
max	40	40	

Game B

Payoff table

	L	R
T	$\underline{320}, 40$	$40, \underline{80}$
B	$40, \underline{80}$	$\underline{80}, 40$

Regret table

	L	R	max
T	$0, 40$	$40, 0$	40
B	$280, 0$	$0, 40$	280
max	40	40	

Game C

Payoff table

	L	R
T	$\underline{44}, 40$	$40, \underline{80}$
B	$40, \underline{80}$	$\underline{80}, 40$

Regret table

	L	R	max
T	$0, 40$	$40, 0$	40
B	$4, 0$	$0, 40$	4
max	40	40	

The traveler's dilemma

Regret of player i for a given strategy profile $(a'_i, a'_j) \in A$ is equal to:

$$R_i(a'_i, a'_2) = \max_{a_i \in A_i} \left(\min(a_i, a'_j) + P(a_i, a'_j)\right) - \left(\min(a'_i, a'_j) + P(a'_i, a'_j)\right)$$

To calculate it we need to consider two cases:

$$\begin{aligned} a'_j &= 180 \Rightarrow R_i(a'_i, 180) = \begin{cases} 180 - 180 = 0 & \text{if} \quad a'_i = 180 \\ 180 - 180 + P = P & \text{if} \quad a'_i > 180 \end{cases} \\ a'_j &> 180 \Rightarrow R_i(a'_i, a'_j) = a'_j - 1 + P - \left(\min(a'_i, a'_j) + P(a'_i, a'_j)\right) \end{aligned}$$

We can summarize it in the form of the table:

	$a'_i = 180$	$a'_i > 180$
$a'_j = 180$	0	P
$a'_j > 180$	$a'_j - 181$	$a'_j - 1 + P - \min(a'_i, a'_j) - P(a'_i, a'_j)$

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The traveler's dilemma

So the maximum regret of player i for a given strategy $a_i' \in A_i$ is equal to:

$$\max \mathrm{regret}_i(a_i') = \begin{cases} 119 & \text{when} \quad a_i' = 180 \\ \max(P, 118) & \text{when} \quad a_i' = 181 \\ \max(2P-1, 0) & \text{when} \quad a_i' \geqslant 182 \end{cases}$$

Now we can solve for the minimax strategies in this game which is summarized in the following table:

Values of P	Minmax regret strat.	Minmax value
$P = 0$	$\{300\}$	0
$P \in \{1, \ldots, 59\}$	$\{300-2P, \ldots, 299, 300\}$	$2P-1$
$P \in \{60, 61, \ldots, 118\}$	$\{181\}$	118
$P = 119$	$\{180, 181\}$	119
$P \in \{120, 121, \ldots\}$	$\{180\}$	119

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For example, for $P = 0$, the only minimax regret strategy is to bid 300. For $P = 5$, any bid in the set $\{290, 291, \ldots, 300\}$ is a minimax regret strategy. For $P = 180$ the only minimax regret strategy is to bid 180.

Choose an effort game

The regret of a given profile of actions $(a'_i, a'_j) \in A$ is given by:

$$\begin{aligned} R_i(a'_i, a'_j) &= \max_{a_i \in A_i} \left(\min(a_i, a'_j) - ca_i\right) - \left(\min(a'_i, a'_j) - ca'_i\right) \\ &= a'_j - ca'_j - \min(a'_i, a'_j) + ca'_i \end{aligned}$$

Maximum regret for a strategy a'_i is given by:

$$\begin{aligned} \max \text{regret}_i(a'_i) &= \max_{a_j \in A_j} \left(a_j - ca_j - \min(a'_i, a_j) + ca'_i\right) \\ &= \max\left(170(1-c) - a'_i, -110c\right) + ca'_i \end{aligned}$$

Minimax regret is given by:

$$\min_{a_i \in A_i} \left[\max\left(170(1-c) - a_i, -110c\right) + ca_i\right]$$

Let's define $a^*_i \in A_i$ as the value of player i strategy for which the two elements of the above max function are equal:

$$170(1-c) - a^*_i = -110c \quad \Rightarrow \quad a^*_i = 170 - c(170 - 110)$$

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Choose an effort

In order to find the minimax regret, it is clear that we need to consider three cases:

$$
\begin{aligned}
a_i' = 110 \quad &\Rightarrow \quad \max \operatorname{regret}_i(110) = (1-c)(170-110) \\
a_i' = a_i^* \quad &\Rightarrow \quad \max \operatorname{regret}_i(a_i^*) = c(1-c)(170-110) \\
a_i' = 170 \quad &\Rightarrow \quad \max \operatorname{regret}_i(170) = c(170-110)
\end{aligned}
$$

Since $c \in (0,1)$ it must be that the minimax regret strategy is $a_i' = a_i^*$ and the minimax regret value is $\max \operatorname{regret}_i(a_i^*) = c(1-c)(170-110)$.
For example if $c = 0.1$, the minimax regret strategy is equal to 164 and if $c = 0.9$, the minimax regret strategy is equal to 116, which is in accordance with the experimental evidence.

An assignment

1. Write a program in Microsoft Excel that calculates minmax regret strategies in the traveller's dilemma game depending on the value of a bonus/malus.
2. Write a program in Microsoft Excel that calculates minmax regret strategies in choose an effort game depending on the cost of effort value.

A coordination game with a secure outside option

Regret table

	L	M	R	max
T	$0, 0$	$180, 90$	$0, 50$	180
B	$90, 180$	$0, 0$	$x, 140$	$\max(90, x)$
max	180	90	140	

where $x \in \mathbb{R}$.
So

- if $x < 180$, then minimax regret strategies are (B, M)
- if $x > 180$, then minimax regret strategies are (T, M)

The Kreps game

Regret table

	L	M	NN	R	max
T	$0, 0$	$10, 5$	$20, 20$	$30, 300$	30
B	$200, 290$	$0, 140$	$0, 10$	$0, 0$	200
max	290	140	20	300	

The only Nash equlibria in pure strategies of this game are (T, L) and (B, R). On the other hand, the only minimax regret pure strategy profile is (T, NN).

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We have developed an application to test departures from Nash-Equilibria.

You may find info here:
`http://www.mlewandowski.waw.pl/game-of-rows/`

And also download the app here:
`https://play.google.com/store/apps/details?id=pl.sgh.gametheory`

Team members (apart from me): Michał Jakubczyk and Bogumił Kamiński

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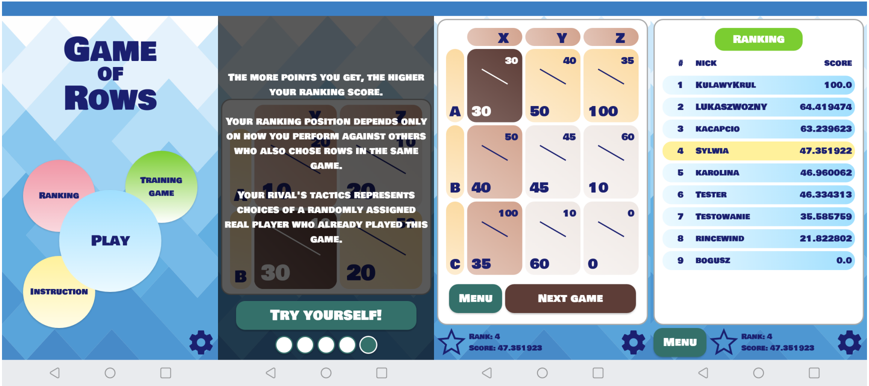
Game of Rows



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