Advanced Microeconomics I - Problem set
Due date: classes on November, 5

Problem 1. *Suppose there are three monetary outcomes: 1 dollar, 2 dollars, 3 dollars. Consider the Marschack-Machina triangle* $\{(p_1, p_3) \in \mathbb{R}^2_+ : p_1 + p_3 \leq 1\}$*, where* p_1, p_3 *denote the probability of receiving 1 and 3 dollars respectively. It is understood that the probability of receiving 2 dollars equals:* $p_2 = 1 - p_1 - p_3$*. A typical lottery in this space will be denoted by* P, Q *or* R*.*

a) *For a given lottery* P *over these outcomes, determine the region of the MM triangle in which lie the lotteries* Q *such that* $\int u(x)dF_Q(x) \geq \int u(x)dF_P(x)$ *holds for every nondecreasing function* $u : \mathbb{R} \to \mathbb{R}$*, where* F_P *is a CDF (cumulative distribution function) of a lottery* P*.*

b) *Given a lottery* P*, determine the region of the MM triangle in which lie the lotteries* Q *such that* $F_Q(x) \leq F_P(x)$ *for every* x*.*

c) *If two lotteries have the same mean, what are their positions relative to each other in the MM triangle.*

d) *Given a lottery* P*, determine the region of the MM triangle in which lie the lotteries* Q *with the same mean such that* $\int u(x)dF_P(x) \geq \int u(x)dF_Q(x)$ *for every nondecreasing and concave functions* $u : \mathbb{R} \to \mathbb{R}$*.*

d) *Given a lottery* P*, determine the region of the MM triangle in which lie the lotteries* Q *which are mean preserving spreads of* P *(to each element of* x *the support of* P *add a* z *distributed according to the CDF* H_x *with* 0 *mean; the resulting reduced lottery is an MPS of* P*).*

e) *Given a lottery* P*, determine the region of the MM triangle in which lie the lotteries* Q *such that* $\int_0^x F_Q(t)dt \geq \int_0^x F_P(t)dt$*, for all* x*.*

Problem 2. *Jan goes on vacation. A new sale is available at the travel agency: Jan draws a coupon with the trip destination from an urn. Jan may decide, however, which of the available two urns to draw from. In urn I there are three "Seychelles" and one "Tenerife" coupons. In urn II, there are two "Seychelles", one "Spitsbergen" and one "Patagonia" coupons. Jan likes both cold outbacks with beautiful nature and clean air, as well as hot islands with lots of tourists. He also likes surprises. It does not bother him, therefore, that he may fly to one island or another or one place in the world or another. However, Jan would at least like to know in advance whether he is going to bask in the sun in some warm place or freeze in the colds somewhere while enjoying the beauties of untouched nature. He therefore prefers to draw from urn I. The agency worker checks his computer and apologizes to Jan saying that the lottery conditions have changed, because the Seychelles' tours have been almost sold out. So the lottery has changed. Urn I (now called urn III) instead of three contains only one and urn II (now called urn IV) instead of two does not contain any Seychelles' coupons. However, to compensate for the limited access to the popular trip to Seychelles, it was decided to add four "Spitsbergen" coupons to both urns. Jan quickly realizes that now urn III contains both cold and warm destinations and urn IV only cold ones. So he decides to choose from urn IV.*

a) *Write down payoff (trips) probabilities in each of the four urns assuming that drawing is fair.*

b) *b) Write down lotteries (urns I IV) in the form of compound lotteries of the following form:*

$$\alpha P + (1 - \alpha)R$$
$$\alpha Q + (1 - \alpha)R$$
$$\beta P + (1 - \beta)S$$
$$\beta Q + (1 - \beta)S$$

so that it could be shown that John's preferences do not satisfy the axiom of independence. Assume that the axiom of independence is given by: $P' \succ Q' \iff \alpha' P' + (1-\alpha')R' \succ \alpha' Q' + (1-\alpha')R'$*, where* P', Q', R' *are any lotteries and* γ *is a number from the interval* $[0,1]$*. Specify the probabilities* α, β *in the fractional form and lotteries* P, Q, R, S *in the form* $(x_1, p_1; ...; x_{n,n})$*, where* x_i *is the payoff and* p_i *the probability of getting it; write* $(x, 1)$ *if the lottery is degenerate.*

c) *c) Show using the lotteries defined above that Jan violates the axiom of independence.*